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Linear Cold Fluid Model of a Free-electron Laser with an Axial Guide Field

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Stimulated backscattered radiation from intense relativistic electron beams has created considerable interest in the past few years, particularly in relation to free electron lasers (FEL), whose potential advantages include, among other things, continuous frequency tunability, high operating power, and high efficiency. The operating mechanism in an FEL is a parametric process in which a long wavelength pump field interacts with a beam of relativistic electrons.[1] The pump field is generated by a static helical magnetic field which scatters a relativistic eletron beam, and the scattered radiation has a wavelength much shorter than the wiggler wavelength and depends on the beam electron energy.

In order for valid application of the fluid theory, the condition $k\lambda_D \ll 1$ should be fulfilled, that is, the system should be in the stimulated Raman scattering regime.[2] This condition requires a cold beam (small Δp), an intense electron beam (I >1 kA) and relatively low electron energy ($\gamma < 10$).[3]

One method for increasing the beam current is by the application of an axial homogeneous magnetic field B_{oz} much stronger than the self-field of the electron beam. Electrons tend to spiral about the guide magnetic field lines, and the electrons drift predominantly in the z-direction, thus increasing the beam current.

IT has been known that the introduction of an axial magnetic field in the FEL configuration[4,5] brings about enhancement in laser gain and efficiency and other interesting effects.[3,6]

In this paper, we investigate the effect of an axial homogeneous magnetic field upon the dispersion relation and the ponderomotive potential which play an important role in the saturation of the instability, and we perform a gain calculation based on the Madey theorem.

For FEL involving an intense electron beam and relatively low electron energy, the collective effects play the major role in the operation; thus the whole process can be described by linear fluid theory.[7]

In this system, a homogeneous axial guide magnetic field is added to a right-handed helical magnetic wiggler,

$$\boldsymbol{B}_o = B_{oz}\hat{e}_z + \delta B_o(\hat{e}_x \cos k_o z + \hat{e}_y \sin k_o z), \tag{1}$$

where B_{oz} and δB_o represent the strength of the guide magnetic field and the wiggler magnetic field, respectively, and k_o is the wavenumber of the wiggler field.

Adopting the Coulomb gauge, the transverse component of the Maxwell wave equation is

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{A}_\perp = -\frac{4\pi}{c}\boldsymbol{J}_\perp. \tag{2}$$

The zero-order response of the electron beam passing through the helical field is[8]

$$v_o = v_{oz}\hat{e}_z + \frac{\omega_{ce}k_o}{\gamma_o K^2}(\hat{e}_x \cos k_o z + \hat{e}_y \sin k_o z) \tag{3}$$

where $\omega_{ce} = \frac{e\delta B_o}{mc}$ is the electron cyclotron frequency due to the helical wiggler field, γ_o is the relativistic factor, and $K^2 = -k_o^2 + \frac{\Omega_{ce}k_o}{\gamma_o v_{oz}} - \frac{\omega_{pe}^2}{\gamma_o c^2}$ where Ω_{ce} is the electron cyclotron frequency due to the axial homogeneous magnetic field.

With linearization of the perturbed quantities, Eq. (2) can be written as

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A'_\perp = \frac{4\pi e}{c}(n_o v'_\perp + n' v_{\perp o}). \tag{4}$$

The perturbed velocity perpendicular to the direction of the beam propagation (z) due to the perturbed electric wave field of the form $e^{-i\omega t}$ in the presence of an axial magnetic field is written as[9]

$$v'_\perp = -\frac{ei}{m}\frac{1}{\omega \mp \Omega_{ce}}E'_\perp \tag{5}$$

were $\mp$ denotes the right-handed and the left-handed rotation, respectively. Hereafter we will consider the right-handed rotation only. Note that $\boldsymbol{E}'_{\perp}$ and $\boldsymbol{A}'$ are related by $E'_{\perp}=-\frac{1}{c}\frac{\partial A'_{\perp}}{\partial t}$.

For a relativistic plasma propagating in the z-direction with v_{oz},

$$\frac{4\pi e n_o}{c}\boldsymbol{v}'_{\perp}=\frac{\omega_{pe}^2}{\gamma_o c^2}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}A'_{\perp}. \tag{6}$$

Thus we can rewrite Eq. (4) as

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\omega_{pe}^2}{\gamma_o c^2}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}\right]A'_{\perp}=\frac{4\pi e n'}{c}v_{\perp o}. \tag{7}$$

Assuming all the perturbed quantities have the form

$$\xi=\sum_k \xi_k e^{i(kz-\omega t)}$$
$$(\xi=A'_x,\ A'_y,\ A'_z,\ E'_x,\ E'_y,\ E'_z,\ n',\ v') \tag{8}$$

and using Eq. (7) with the linearized continuity equation with Poisson's equation yields

$$A'_{xk}=\frac{2\pi ec\,\omega_{ce}k_o}{\gamma_o K^2}\left(n'_{k+k_o}+n'_{k-k_o}\right)\left[\omega^2-k^2c^2-\frac{\omega_{pe}^2}{\gamma_o}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}\right]^{-1}, \tag{9}$$

$$A'_{yk}=-\frac{i2\pi ec\,\omega_{ce}k_o}{\gamma_o K^2}\left(n'_{k+k_o}-n'_{k-k_o}\right)\left[\omega^2-k^2c^2-\frac{\omega_{pe}^2}{\gamma_o}\frac{\omega-kv_{oz}}{\gamma-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}\right]^{-1}, \tag{10}$$

$$E'_{z,k+k_o}=\frac{4\pi i e n_o}{\omega-(k+k_o)v_{oz}}v'_{z,k+k_o}. \tag{11}$$

Then the amplitude of the electromagnetic wave can be written in terms of the amplitude of the electrostatic wave,

$$E'_{xk}=-\frac{\omega_{ce}}{2\gamma_o\omega}\frac{k_o(k+k_o)}{K^2\epsilon_{em}(k,\omega)}E'_{z,k+k_o} \tag{12}$$

where

$$\epsilon_{em}(k,\omega)=\frac{k^2c^2}{\omega^2}+\frac{\omega_{pe}^2}{\gamma_o\omega^2}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}-1$$

is the electromagnetic dielectric function.

This phenomenon involves a three-wave parametric process in which a scattered electromagnetic wave grows as a result of the interaction between a pump wave (a periodic magnetostatic ripple in the laboratory frame) and a negative energy beam collective mode (the space charge wave). The term which drives the scattered wave is the nonlinear currents, and the idler (plasma) is again excited through the ponderomotive interaction. This subsequent excitation of the idler wave amplifies the scattered wave through the coupling with the pump wave, and thus the stimulated scattering process occurs.

The electromagnetic wave to be amplified is chosen to be right-hand polarized and is given by

$$\boldsymbol{E}^l=E_l\left[\hat{e}_x\cos(kz-\omega t+\phi)-\hat{e}_y\sin(kz-\omega t+\phi)\right],$$
$$\boldsymbol{B}^l=B_l\left[\hat{e}_x\sin(kz-\omega t+\phi)+\hat{e}_y\cos(kz-\omega t+\phi)\right], \tag{13}$$

where ϕ is the phase angle between the electron and the electromagnetic wave.

Relating these with the notation introduced beforehand, we can write

$$A'_x=E_l\frac{c}{\omega}\sin(kz-\omega t+\phi),$$
$$A'_y=E_l\frac{c}{\omega}\cos(kz-\omega t+\phi). \tag{14}$$

Due to the presence of the EM wave, the total linear response of the electron beam to the applied field is

$$v=\left[1-\frac{1}{\gamma_o^2}-\left(\frac{\omega_{ce}k_o}{c\gamma_o}\right)^2\left(k_o^2+\frac{\omega_{pe}^2}{\gamma_o c^2}\right)^{-2}\right]^{1/2}c\hat{e}_z+\left[\frac{eE_l}{m\omega\gamma_o}\sin(kz-\omega t+\phi)+\frac{\omega_{ce}k_o}{\gamma_o K^2}\cos k_o z\right]\hat{e}_x+\left[\frac{eE_l}{m\omega\gamma_o}\cos(kz-\omega t+\phi)+\frac{\omega_{ce}k_o}{\gamma_o K^2}\sin k_o z\right]\hat{e}_y. \tag{15}$$

The ponderomotive force can be calculated as

$$F_z=\frac{e\omega_{ce}E_l}{c\gamma_o}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_o K^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right]\cos\left[(k+k_o)z-\omega t+\phi\right] \tag{16}$$

and the ponderomotive potential is

$$\phi(z)=-\frac{e\omega_{ce}E_l}{c\gamma_o(k+k_o)}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_o K^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right]\sin\left[(k+k_o)z-\omega t+\phi\right].$$

In the cold beam, weak pump, Raman regime, the dominant saturation mechanism appears to be electron trapping in the electrostatic idler wave.[10] However, in the later stage of operation when the EM wave is amplified to some extent, the saturation of the instability is due to electron trapping by the ponderomotive potential.

The longitudiual component of the equation of motion can be written as

$$\gamma_o^3 m\left(\frac{\partial}{\partial t}+v_{oz}\frac{\partial}{\partial z}\right)v_z'=-eE_z'+F_z \tag{18}$$

where E_z' is the electrostatic wave field described in Eq. (11) and F_z is the ponderomotive force.
Substituting the Poisson equation in Eq. (9), we obtain

$$A_{xk}'=-\frac{ic\,\omega_{ce}k_o(k+k_o)}{2K^2\gamma_o}\left[\omega^2-k^2c^2-\frac{\omega_{pe}^2}{\gamma_o}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}\right]^{-1}E_{z,k+k_o}'. \tag{19}$$

From Eq. (14) and Eq. (19) we write

$$E_z'=E_o'\cos[(k+k_o)z-\omega t+\phi] \tag{20}$$

where

$$E_o'=-\frac{2E_lK^2\gamma_o\omega}{\omega_{ce}(k+k_o)k_o}\epsilon_{em}(k,\ \omega).$$

Thus, we rewrite Eq. (18) as

$$\gamma_o^3 m\left(\frac{\partial}{\partial t}+v_{oz}\frac{\partial}{\partial z}\right)v_z'=-eE_o\cos[(k+k_o)z-\omega t+\phi] \tag{21}$$

where

$$E_o=E_o'-\frac{\omega_{ce}E_l}{c\gamma_o}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_oK^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right].$$

The solution to Eq. (21) is

$$v_z'=\frac{eE_o}{m\gamma_o^3}\frac{\sin[(k+k_o)z-\omega t+\phi]}{\omega-(k+k_o)v_{oz}}. \tag{22}$$

Substituting Eq. (22) in Eq. (11), and after some manipulation, we finally obtain the dispersion relation from which the frequency and the growth rate in the linear regime can be obtained;

$$\left\{[\omega-(k+k_o)v_{oz}]^2-\frac{\omega_{pe}^2}{\gamma_o^3}\right\}\left(\omega^2-k^2c^2-\frac{\omega_{pe}^2}{\gamma_o}\frac{\omega-kv_{oz}}{\omega-kv_{oz}-\frac{\Omega_{ce}}{\gamma_o}}\right)=-\frac{\omega_{pe}^2\omega_{ce}^2(k+k_o)k_o\omega}{2K^2c\gamma_o^5}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_oK^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right]. \tag{23}$$

Note that by the application of an axial guide magnetic field the coupling term is changed by the factor

$$-\frac{\omega}{K^2c}\frac{k_o^3}{k+k_o}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_oK^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right]$$

compared to that[8] without the axial field, and Eq. (23) is the limiting case $(k\lambda_D\ll 1)$ of the dispersion relation obtained by T. Kwan and J. M. Dawson.[2]

If our system is assumed to be in the regime where the ponderomotive potential dominates the trapping process, the introduction of the axial magnetic field results in a reduction of the maximum amplitude of the potential well, ϕ, by a factor of

$$\frac{kk_o}{k+k_o}\left[\left(1+\frac{\omega_{pe}^2}{\gamma_oK^2C^2}\right)\frac{1}{k}-k_oK^{-2}\right]$$

compared to that[8] without a guide field. In Fig. 1, this ratio is plotted as function of wavenumber for beam energy $\gamma_o=2$. For the system parameters used, $\Omega_{ce}=7.0\ \omega_{pe}-11.0\ \omega_{pe}$ and $\omega_{ce}=0.7\ \omega_{pe}$, the ratios are between 0.15 and 0.18.

Particles followsing closed orbits in the phase space in the reference frame co-moving with the ponderomotive potential are trapped by the potential well and they satisfy the inequality $\phi(z)\geq W$, where $W=\frac{1}{2}mv_{oz}'^2$ is the kinetic energy of the electron in the moving frame. Thus, in most situations of experimental interest almost all the electrons are trapped by the ponderomotive potential. Many different periods of oscillation in various orbits cause rapid phase mixing, thereby causing instability saturation at an early stage. The axial homogeneous magnetic field in the FEL configuration alleviates this tendency by reducing the maximum amplitude of the potential well. The reduction in the amplitude of the ponderomotive potential coherently and effectively decelerates the electrons; thus the efficiency of energy extraction is greatly enhanced.[8]

The Lorents equation is written as

$$mc^2\frac{d\gamma}{dt}=-e\boldsymbol{E}\cdot\boldsymbol{v} \tag{24}$$

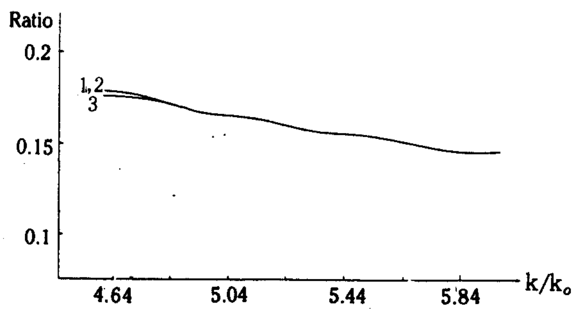

Fig. 1. The ratio of a ponderomotive potential with an axial guide field to one without a guide field as a function of wave number for beam energy $\gamma_o=2$.
Lines (1), (2), and (3) represent $\Omega_{ce}=7.0\ \omega_{pe}$, $\Omega_{ce}=9.0\ \omega_{pe}$, and $\Omega_{ce}=11.0\ \omega_{pe}$, respectively ($\omega_{ce}=0.7\ \omega_{pe}$).

where $\boldsymbol{E}$ is the electric field of the electromagnetic wave of Eq. (13).

The first-order perturbation equation of γ caused by the electronagnetic wave is

$$mc^2\frac{d\gamma_1}{dt} = -e\boldsymbol{E}^1\cdot\boldsymbol{v}_o$$
$$= -\frac{eE_1\omega_{ce}k_o}{\gamma_o K^2}\cos\{(k+k_o)z-\omega t+\phi\}. \quad (25)$$

Integrating Eq. (25) in the interaction region [O,L], we have

$$\gamma_1 = -\frac{eE_1\omega_{ce}k_o}{\gamma_o K^2 mc^2 v_{oz}}\frac{\sin(\Delta kL+\phi)-\sin\phi}{\Delta k} \quad (26)$$

where $\Delta k = (k+k_o)-\frac{\omega}{v_{oz}}$.

Squaring γ_1 and averaging over ϕ gives

$$<\gamma_1^2> = \frac{L^2}{2}\left(\frac{eE_1\omega_{ce}k_o}{\gamma_o K^2 mc^2 v_{oz}}\right)^2\frac{\sin^2 U}{U^2} \quad (27)$$

where $U=\frac{L}{2}\Delta k$.

According to the Madey theorem,[11] we obtain a second-order perturbation of γ,

$$<\gamma_2> = \frac{1}{2}\frac{d}{d\gamma}<\gamma_1^2>. \quad (28)$$

The gain is written as[12]

$$G = -\frac{<\gamma_2>mc^2 I/e}{\frac{1}{2}\varepsilon_o E_1^2 cA_\omega} \quad (29)$$

where A_ω is the area of the optical wave and I is the electron beam current.

The final form for gain is

$$G = -\frac{1}{4}\frac{I_e}{m\varepsilon_o}\frac{\omega_{ce}^2 k_o^2}{\gamma_o^2 K^4}\frac{L^2}{A_\omega}\frac{kL}{v_{oz}^5\gamma_o^3}\frac{d}{dU}\frac{\sin^2 U}{U^2}. \quad (30)$$

As can be noted in Eq. (3), the use of an axial guide field permits a high undulatory transverse velocity of the unperturbed electron beam at lower values of the pump field.[5] This effect contributes to gain enhancement according to Eq. (27).

In conclusion, the presence of an axial guide field leads to gain enhancement both by reducing the maximum amplitude of the ponderomotive potential and by increasing the transverse velocity of the electron beam.

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축방향 유도 자기장을 가진 자유전자 레이저의 선형 찬유체 모형

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(1988년 2월 10일 받음)

축방향 유도 자기장을 가진 자유전자 레이저의 분산관계와 ponderomotive 퍼텐셜을 찬유체 선형 모형으로 간략하게 유도하였고, 이 유도 자기장이 레이저 이득과 효율에 미치는 영향을 논의하였다.