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Period Doubling and the Chaotic Region in RF-excited Plasmas

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The period-doubling and concomitant subharmonic generation are numerically investigated for RF-excited plasmas. Due to the plasma nonlinear capacitance, the current and the voltage waveforms are found to contain higher harmonic components and subharmonic components. Above a certain power level of the driver, subharmonics which are indicative of a period-doubling(PD) bifurcation are found to become dominant. The equivalent circuit equation modeling the nonlinear capacitance of the plasma is solved numerically by using the Runge-Kutta-Gill method. The period-doubling and the chaotic region are explored by varying the driver amplitude as well as the driver frequency.

It has been known that a driven nonlinear RLC circuit can exhibit period-doubling (PD) and chaos. The nonlinear element in the circuit is a varacter diode whose capacitance varies as a function of the voltage across it [1]. A capacitively coupled RF discharge system also displays a similar characteristic that of a nonlinear RLC circuit [2–4]. In this case, the nonlinear capacitance due to plasma sheaths is known to be the cause of the period-doubling and the bifurcations. The electrical characterization of an RF plasma processing system is quite important to understand the power dissipation mechanism and to control the ion bombardment.

The chaotic behavior of an RF discharge system is determined by the driver frequency, the applied RF voltage, and the gas pressure, or by the feedback term of the time delay [3]. Such chaotic behavior is classified as driven chaos. In this study, we investigate the effects of the driver frequency, the RF voltage, and the external circuit parameters on the characteristics of a driven chaos system by using a numerical calculation of the equivalent circuit equation. Also, we physically explain to the formulation of the equivalent circuit equation.

When an electrical system with nonlinear impedance characteristics operates periodically with frequency f, higher harmonics will be generated by the nonlinearity at frequencies $2f$, $3f$, $4f$, *etc.*, and subharmonics will also be generated at frequencies $f/2$, $3f/2$, $5f/2$, *etc.* Above a certain power level of the driver, subharmonics which are indicative of a period-doubling (PD) bifurcation become dominant [2–4]. A further increase in the RF voltage level beyond the PD threshold can lead to successive stages of period multiplication that result in the onset of chaos at a finite excitation level. Period-doubling bifurcations manifest themselves in a driven system by an alternation of state variables between different values on successive cycles of the excitation waveforms. This corresponds to the generation of subharmonics ($f/2$) and odd-multiples of the half-harmonic frequency.

Figure 1 is an equivalent circuit diagram of the RF plasma system. One electrode is powered, and the other is grounded. The plasma RF chamber is represented by a parallel tuned circuit with L_p, C_p, and R_p, while the resistance and the capacitance of both sheaths(inner and outer) are represented by R_s and C_s, respectively. In addition to these, the external circuit values of C_{ext}, L_{ext}, and R_{ext} contributes to the equivalent circuit parameters. The plasma sheath capacitance is actually comprised of two sheaths (inner and outer capacitances) in series. In the usual circumstances, the plasma sheath capacitance due to the sheath at the powered electrode dominates.

This circuit has two resonance frequencies [4]. The first is the resonance frequency of the parallel circuit $\omega_1 = (L_pC_p)^{-1/2}$ (which is equal to the electron plasma frequency). The second is $\omega_2 = (LC_{eq})^{-1/2}$ where L and C_{eq} are the equivalent inductance and capacitance of the whole system, respectively. The equivalent capacitance of the chamber, C_{ch}, representing the RF chamber (the bulk plasma and the sheath) is a complex function of the driver frequency ($w = 2\pi f$), the plasma resistance (R_p), the sheath capacitance (C_s), and the cell capacitance (C_p) and is given by [4]

$$C_{ch} = C_s \frac{1}{1 - Q_2 C_s / C_p} \tag{1}$$

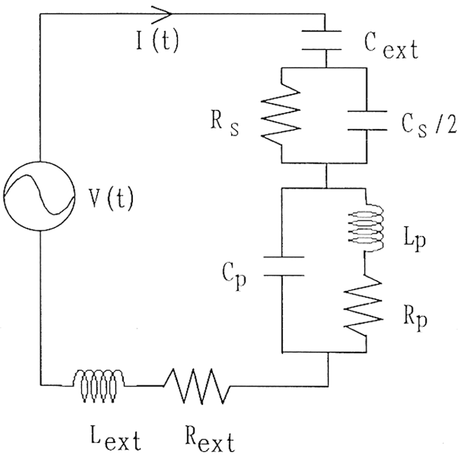

Fig. 1. The equivalent circuit representing the RF chamber and the external elements.

where

$$Q_2 = \frac{\omega_1^2/\nu_{eff}^2 - 1}{1 + \omega_1^4/\omega^2\nu_{eff}^2}.$$

Here, the effective collision frequency (ν_{eff}) is the sum of the electron-neutral collision frequency (ν) and the stochastic heating frequency (ν_{st}).

Combining with the external capacitance, the capacitance of the equivalent circuit is written as

$$C_{eq} = \frac{C_{ext}}{1 + \frac{C_{ext}}{C_{ch}}}. \tag{2}$$

Thus, the charge Q on the capacitor obeys

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C_{eq}} = V(t) \tag{3}$$

where $V(t) = V_{rf}\cos(2\pi f t)$ is the applied voltage and R is the equivalent resistance of the whole system.

Since the sheath capacitance depends on the control parameters in a complicated way, its dependence on the applied RF voltage is not simple [5]. There have been some assumptions concering the functional dependence of C_{eq} on $V(t)$. One of them is [3]

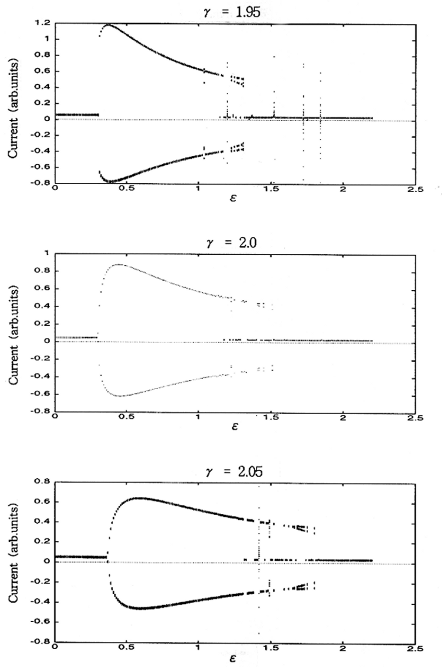

Fig. 2. The variation of the normalized current with the driver amplitude for three different values of the driver frequency around $\gamma = 2.0$.

$$C_{eq} = \frac{C_1}{1 + kV_c} \tag{4}$$

where C_1 is a constant capacitance, k is a constant, and V_c is the voltage across the capacitor C_{eq}. Comparing Eq. (4) with Eq. (2), we note that the dependence of C_{ch} on the driver frequency is neglected in this assumption. If we assume $Q_2 \ll C_p/C_s$, then we have $C_{ch} \simeq C_s$. Along with these, Eq. (4) assumes $C_s \sim \frac{1}{V_c}$. This is a simple-minded approximation. Since the sheath capacitance is inversely proportional to the sheath width, which is a nonlinear function of the applied rf voltage [5,6], we treat

Table 1. Values of parameter.

	Symbol	Values	Normalized Symbol	Values
RF frequency	f	12.86 MHz	γ	1.8
		13.39 MHz		1.95
		13.56 MHz		2.0
		13.90 MHz		2.05
Inductance	L	7.6 μH		
Capacitance	C_1	72.50 pF		
Resistance	R	104.8 Ω	α	0.1
Time step	Δt	10^{-9} sec	τ	4.2601 $\times$ 10^{-2}
Driver amplitude	V_{rf}		ϵ	0 - 2.2

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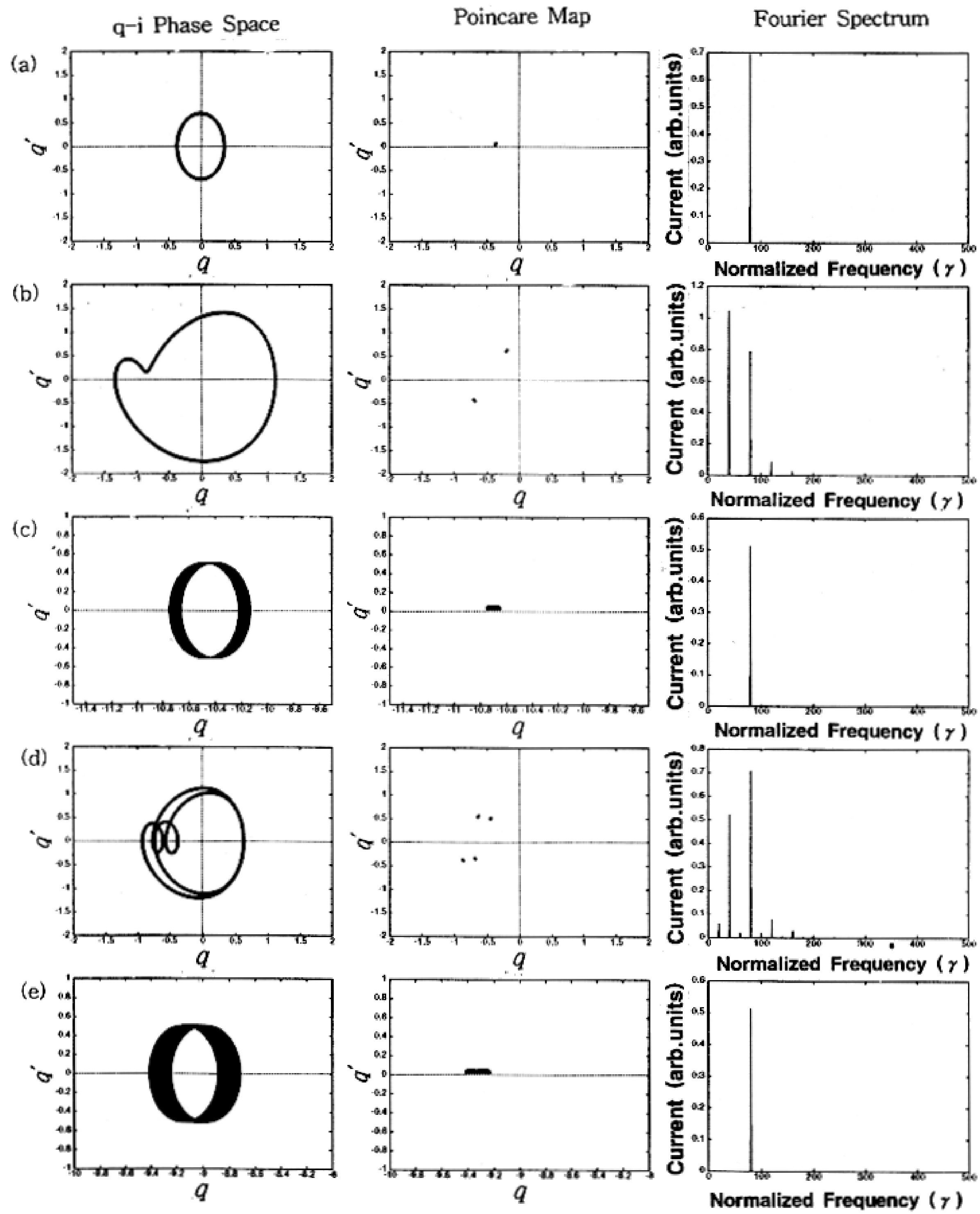

Fig. 3. The phase-space diagrams (q, q'), the poincare maps, and their Fourier spectra for various driver amplitudes. Here, γ=1.95, α=0.1: (a) ϵ=0.2, (b) ϵ=0.5, (c) ϵ=1.2, (d) ϵ=1.23, and (e) ϵ=1.3.

this nonlinearity in an alternative formula as

$$C_{eq} = C_1 e^{-kV_c}. \quad (5)$$

Using the normalization

$$q = \frac{Q}{C_1 V_{rf}}, \qquad \tau = t\sqrt{\frac{1}{LC_1}}, \qquad \gamma = 2\pi f\sqrt{LC_1}$$

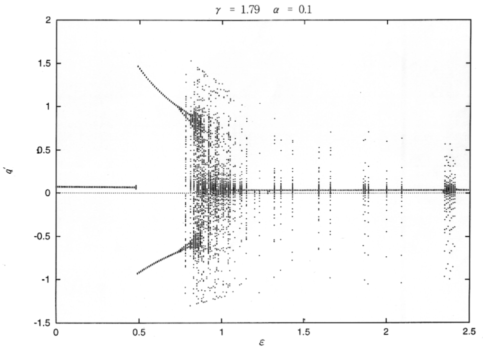

Fig. 4. The variation of the normalized current with the driver amplitude for the driver frequency γ=1.79 and the resistance α=0.1.

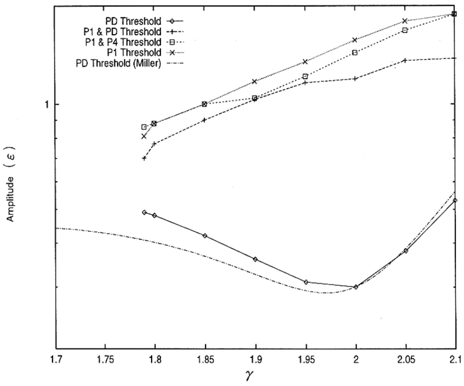

Fig. 6. The threshold driver amplitude (ϵ) for period-doubling as a function of the driver frequency (γ).

$$\alpha = R\sqrt{\frac{C_1}{L}}, \qquad \epsilon = kV_{rf},$$

Eq. (3) with Eq. (5) can be written in a dimensionless form as

$$q'' + \alpha q' + qe^{\epsilon q} = \cos(\gamma\tau), \tag{6}$$

where the prime denotes differentiation with respect to τ.

This nonlinear equation gives rise to the period-doubling. If V_{rf} is greater than some threshold value, the solution of Eq. (6) has subharmonics (the multiples of $f/2$); in other words, the period-doubling phenomenon can happen in the discharge current. The value of a period-doubling threshold is a complex function of L, R, C_p, C_{ext}, C_s, and f. The gas pressure, p, may be related to the system impedance L, R, C_p, and C_s. The analytic theory [3] indicates that the lowest PD threshold occurs at $f = 1/\pi\sqrt{LC_{eq}}$, that is, two times the resonant frequency of the system ($\omega_2/2\pi$). In other words, the driver frequency which doubles the resonance frequency of the system can easily result in PD.

The formation of the plasma sheath is considered to be associated with the difference of motions between highly mobile electrons and heavy positive ions. The sheath capacitance is inversely proportional to the sheath thickness [5]. The nonlinear sheath capacitance may be explained alternatively by taking into account the possibility of a sheath instability which arises when the ion transit time through the sheath is comparable to the RF period [7].

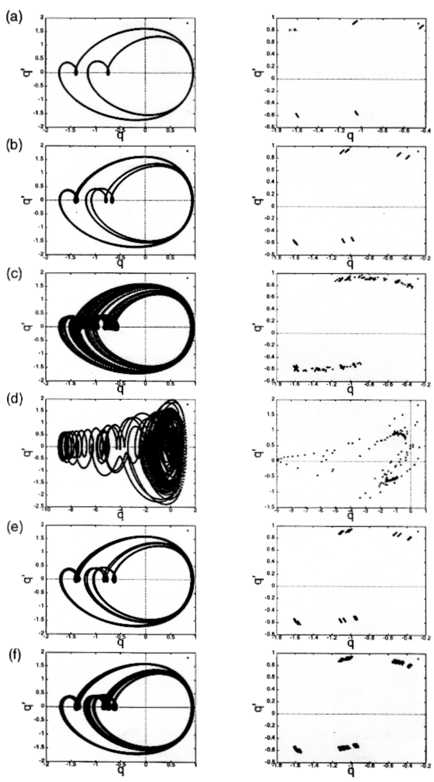

Fig. 5. The phase-space diagrams (q, q') and the poincare maps for various driver amplitudes around ϵ=0.8. A slight increase in ϵ results in chaotic behavior: (a) ϵ=0.80, (b) ϵ=0.81, (c) ϵ=0.815, (d) ϵ=0.817, (e) ϵ=0.818, and (f) ϵ=0.82.

In this study, the period-doubling characteristics are investigated for an Ar glow plasma generated by an RF discharge with varying normalized driver amplitudes (ϵ) and driver frequencies (γ) (in a large range around the resonance frequency of the equivalent circuit). The results obtained are presented in a kind of phase diagram in two-dimensional parameter space (γ, ϵ).

Figure 2 shows the bifurcation diagrams calculated by solving Eq. (6) numerically using the Runge-Kutta-Gill method. Three values of the driver frequencies, γ=1.95, 2.0, and 2.05, are considered. The driver amplitude ϵ is increased in steps of 0.01. Raising ϵ, one sees the bifurcation sequence.

Figure 3 shows the phase-space diagrams (q, q'), the poincare maps, and their Fourier spectra for various driver amplitudes. The driver amplitude increases from (a) to (e). By gentely raising the driver amplitude, it is possible to see a very clear period-doubling sequence. By further increasing the driver amplitude, one observes additional bifurcations, chaos, and then an alternation of windows of periodic and chaotic motions.

In Fig. 4, the variation of the normalized current with the driver amplitude for the driver frequency γ=1.79 and the resistance α=0.1 is shown in a finer grid in ϵ around ϵ=1. In Fig. 5, the phase-space diagrams (q, q') and the poincare maps for various driver amplitudes around ϵ=0.8 are shown. We note that a slight increase in ϵ results in chaotic behavior.

The results of the calculation are summarized in the (γ, ϵ) phase diagram in Fig. 6. The diagram shows the kind of oscillation for each value of the driver frequency and the driver amplitude. For example, taking a driver frequency of γ=2, one finds the following behavior: For a small value of ϵ, the system oscillates with the same period as the driver. For increasing values of ϵ, the system suddenly oscillates with a double period. With a further increase of ϵ, we find the mixed state of P1 (period 1) and PD. Above a certain level of ϵ, we observe the mixed state of P1 and P4. If we investigate these windows on a finer grid scale, we find a normal Feigenbaum period-doubling cascade 4,8,16,$\cdots$, ending in the chaotic region. Finally, we reach the P1 state again. This agrees with our previous study [8] using a particle-in-cell simulation in which the PD domain in the *pd* (pressure times electrode gap distance) vs. period-doubling threshold-voltage curve has several windows in which PD is not found.

In conclusion, the nonlinear characteristics of an RF glow-discharge Ar plasma around a driver frequency of 13.56 MHz are studied by solving the equivalent circuit equation. The calculations determine a threshold voltage ϵ for a given γ and α. The nonlinearity of the plasma-sheath capacitance is the cause of PD in the RF discharge system. It is observed that a further increase in the RF voltage level beyond the PD threshold can lead to successive stages of period multiplication that result in the onset of chaos at a finite excitation level. The identification of possible routes to chaos is also an interesting topic. It is also noted that PD threshold measurements provide a novel diagnostics for determining the plasma sheath capacitance. The modeling of the functional form of the equivalent capacitance, as in Eq. (5), provides fairly good results which are in agreement with those form a particle-in-cell (PIC) simulation [8]. Further study on an accurate model for the dependence of the sheath capacitance on the applied RF voltage and the driver frequency should be pursued.

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