Wiggler-free FEL with an intense helical beam

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The cyclotron autoresonance maser (CARM) is a classical device that uses a beam of gyrating electrons passing through a magnetic field to amplify electromagnetic radiation by the cyclotron maser instability [1]. In the present work, we examine the nonlinear aspects of the CARM interaction with space charge effects. For space charge to be important, the electron beam should be gyrophase coherent (i.e., a helical beam). The random-gyrophase (i.e., gyrotropic) beam does not interact with the space charge wave. The helical (gyrotropic) beam consists of electrons which have the same (arbitrary) azimuthal angles in the plane perpendicular to the velocity space. A detailed description is found in Refs. [2–6].

We use the nonlinear differential equations [2]:

\[ \frac{d^2}{dz^2} \frac{\omega_p^2}{c^2} \beta_{z0} \left( \frac{\nu_z \cos \psi - \nu_x \sin \psi}{|\nu_z|} \right) \delta \alpha = \frac{\omega_p^2}{c^2} \beta_{z0} \left( \frac{\nu_x \sin \psi - \nu_y \cos \psi}{|\nu_x|} \right), \]

(1)

\[ 2k_z^{1/2} \frac{d}{dz} \left( k_z^{1/2} \delta \alpha \right) = -\frac{\omega_p^2}{c^2} \beta_{z0} \left( \frac{\nu_x \sin \psi - \nu_y \cos \psi}{|\nu_x|} \right), \]

(2)

\[ \left( \frac{d^2}{dz^2} - k^2 \right) \delta \phi = \frac{2 \omega_p^2}{c^2} \beta_{z0} \left( \frac{\cos \psi_{sc}}{|\nu_z|} \right), \]

(3)

\[ 2k_z^{1/2} \frac{d}{dz} \left( k_z^{1/2} \delta \phi \right) = -\frac{2 \omega_p^2}{c^2} \beta_{z0} \left( \frac{\sin \psi_{sc}}{|\nu_z|} \right), \]

(4)

\[ u_z \frac{d u_z}{d \xi} = -\Omega_0 u_x / \omega + \delta a (\gamma - n_x u_z) \sin \psi \]

\[ + \frac{d \delta a}{dz} u_z \cos \psi, \]

(5)

\[ \frac{d u_x}{d \xi} = \frac{\omega_0}{\omega} u_x / \omega + \delta a (\gamma - n_x u_z) \cos \psi \]

\[ - \frac{d \delta a}{d \xi} u_z \sin \psi, \]

(6)

\[ u_z \frac{d u_z}{d \xi} = \delta a \left[ k_z \left( u_x \sin \psi + u_y \cos \psi \right) \right] \]

\[ - \frac{d \delta a}{d \xi} \left( u_x \cos \psi - u_y \sin \psi \right) \]

\[ + \gamma \delta \phi \left( \frac{d \delta \phi}{d \xi} \cos \psi_{sc} - n_x \sin \psi_{sc} \right), \]

(7)

where the symbols are the same as in Ref. [2].

To validate the linear aspects of the nonlinear simulation results, we need a linear dispersion relation from an independent theory. We assume a single k mode and a cold electron distribution function, where

\[ F = 2 \pi u \delta (u_z - u_0), \]

and \( \zeta = \alpha + (\Omega / u_z) z, \) \( g = 1[\delta (\zeta / 2 \pi)] \) for a gyrotropic (helical) beam.

Through linearization of the Vlasov equation, Maxwell’s equations, and the continuity equation, the dispersion relation for a helical beam ECM (electron cyclotron maser) is given by

\[ \left( \omega - K v_r \right)^2 - \frac{\omega_p^2}{\gamma (1 - \beta_z^2)} \]

\[ \times \left[ \omega^2 - k^2 c^2 - \frac{\omega_0^2}{\gamma \omega - k v_z - \Omega} \right] \]

\[ + \frac{1}{2} \gamma^2 \beta_z^2 \frac{\omega_0^2}{\gamma \omega - k v_z - \Omega} \]

\[ = \frac{1}{2} \gamma^2 \beta_z^2 \frac{\omega^2 - k^2 c^2}{\gamma \omega - k v_z - \Omega}, \]

(8)

where \( K = k + \Omega / v_z, \) \( \beta_z = v_z / c, \) and \( \Omega = \Omega_0 / \gamma. \)
The initial conditions for the simulations are $\gamma_0 = 1.8$ (corresponding to 400 keV), beam current of 500 A, beam density of $3.32 \times 10^{12}$ cm$^{-2}$, uniform axial magnetic field of 0.34 T, and beam perpendicular velocity of 0.4c (pitch of 0.549). Figs. 1 and 2 summarize the nonlinear simulation runs with varying wave phase velocities (or varying values of $k$ on the horizontal axis). Fig. 1 shows the comparison of the linear growth rates measured in our nonlinear simulations with those of the independent linear theory. The solid line is the helical beam case where the space charge effect and the dotted line is the gyrotrropic beam case without the space charge effect; both are from the linear dispersion relation. The crosses (the circles) are the nonlinear simulation results with (without) the space charge effect. The normalized phase velocity varies from 0.98 to 1.4. For $\beta_\phi = 1$, $kv_z/\Omega$ is approximately 2.689 in the cyclotron mode. Fig. 2 shows the efficiency with the space charge effects in the autoresonance regime. In the slow wave region, the space charge effect reduces the efficiency and the gain.

Generally in the helical beam CARM the space charge effect enhances the growth rate for $\beta_\phi > 1$, but stabilizes it for $\beta_\phi < 1$. The gyrotrropic beam CARM is stable at $\beta_\phi = 1$. In the random gyro-phased (gyrotrropic) beam case, the space charge effect vanishes. The efficiency changes little with the space charge term for the gyrotrropic beam CARM, which is in agreement with Ref. [7].

In summary, the space charge effect is pronounced in the autoresonance regime where $\beta_\phi \sim 1$ and $kv_z/\Omega_0 \sim 2.7$. The nonlinear results from our simulation are consistent with the linear results from a kinetic theory.

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References